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Full wave solution and simulations of laser pulse amplification

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The need for low cost, compact, high- power laser systems with their applications in medicine and high energy physics is growing rapidly. Counter propagating laser pulses amplification promise a breakthrough by the use of much smaller amplifying media, that is, millimeter plasma scale. The full-wave solution for the two laser pulses interact in almost homogenous or plasma channel is conducted along with particle-in-cell simulation for the same pulses' parameters. Motivated by the promise of reduced cost and complexity of the intense lasers, the amplitudes of laser pulses are taken to be small ($a_0 < 1$). The growth rate of the seed pulse and the dephasing limitations are calculated. The results show that the energy is transferred from the pump pulse to the seed pulse effectively depending on the length of amplification and the isolation of the limiting conditions. A wide variety of system parameters such as frequency of laser pulses, plasma density matched to three waves interaction, and intensity of the pump wave and seed wave are studied. The influence of plasma and pulses parameters on simulation results are thoroughly investigated using a moving window technique and are compared with theoretical and numerical predictions. The comparison shows that the numerical full wave solution is very sensitive to any plasma density changes near the entrance of the pump pulse into the plasma.

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1. INTRODUCTION

The necessity of laser pulses with high energy is continuing for high-intensity laser applications such as inertial confinement fusion (ICF), laser Wakefield acceleration (LWFA), and X-ray lasers to name but a few. However, the laser intensity obtained by amplifiers using chirp pulse amplification (CPA) is limited to power of (10^{12} W/ cm^2) by the limitations on the size of diffraction gratings. Recently, a new method for obtaining high laser power above (10^{15} W/ cm^2) has been proposed which uses the Raman backscatter of a pump laser in a plasma to amplify a counter-propagating laser pulse. Dealing with this kind of parametric instabilities such as Stimulated Raman Scattering (SRS) involving solution of some system of second order boundary value differential equations with two resonance points at two different plasma densities and two different reflection points at two different critical plasma densities for electromagnetic laser wave and electrostatic plasma wave.

In this study, we solve the full wave propagation of the laser pump wave, with energy above the threshold of the instability but lower than the relativistic limit, through an inhomogeneous plasma slab with preformed plasma channel immersed in this plasma. The amplified seed wave is supposed to counter-propagating the laser pump wave in the plasma in a one-dimensional problem.

Stimulated Raman scattering is the decay of an electromagnetic wave (the laser) into a scattered electro-magnetic wave and an electron plasma wave.

In some of the applications of (SRS), such as particle acceleration and current-drive in Tokamaks, Raman scattering is beneficial, and it is desirable to drive large amplitude electron plasma waves to increase efficiency. In other applications such as inertial confinement fusion and X-ray lasers, stimulated Raman scattering is detrimental, and it is important to limit the amplitude that the electron plasma waves are driven. It is therefore important to understand the saturation mechanisms which limit the growth of these parametric instabilities as a first step in the eventual process of controlling the amplitude to which these processes grow [Baker (1996)]. It is found that [Kruer (1988)] the highest growth rate for (SRS) can be achieved if the plasma is infinitely homogenous. Thus, it is important for laser pulse amplification to create a homogenous plasma channel as long as possible in the plasma. Because (SRS) is a resonance phenomenon, avoiding any mismatching conditions will be a great success. There are two very important causes of mismatching in the case of (SRS); dephasing and relativistic effects. In this work the plasma channel is taken to be a

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finite near homogenous part of the linearly inhomogeneous plasma. Also, the laser power will be taken to be much lower than the relativistic limit.
In other applications as particle acceleration, an electron plasma wave with a phase velocity close to the speed of light is used to accelerate electrons to very high energy. The electrons traveling slightly slower than the wave see a nearly constant potential in the wave frame that allows them to be efficiently accelerated [Alireza Heidari (2012); Amiranoff (1992)]. For current drive in Tokamaks, Raman scattering is used to drive an electron plasma wave with a phase velocity several times the thermal velocity of electrons in a plasma. In this application near thermal electrons in the plasma are accelerated by the electron plasma wave in a preferential direction, generating a net current around the Tokamak. All of these applications are limited by the amplitude to which the electron plasma wave can be driven, and an understanding of these applications requires detailed knowledge of the saturation mechanisms responsible for limiting the amplitude of these waves [HAFZ (2008); Joshi and Malka (2010)].

2. Outline of The Model

The three wave interaction between counter propagating electromagnetic waves with frequencies and wave numbers (ω_1, k_1) and (ω_2, k_2) and a plasma wave where the frequency and wave number of the plasma wave satisfy the following resonance conditions [TAJIMA and DAWSON (1979); Lehmann and Spatschek (2011)].

$$\omega_p = \omega_1 - \omega_2$$
$$k_p = k_1 - k_2 \quad (1)$$

Where $\omega_p = (\omega_{po}^2 + 3v_t^2 k_p^2)^{1/2}$ is the plasma wave frequency including the thermal corrections with v_t is the thermal velocity of the electrons? The laser pulse exerts a periodic nonlinear force (the pondermotive force) on the electrons and resonantly drive the plasma wave which consists of regions of space charge.
The equations describing the behavior of the electrons are the continuity (Poisson's) equation, the hydromagnetic equation of motion and Maxwell's equations.

$$\partial n/\partial t + \underline{\nabla} \,.\, (n\underline{u}) = 0 \quad (2)$$

$$\partial\underline{u}/\partial t = -(e/m)\underline{E} - \underline{\nabla}\,(u^2/2) - \underline{\nabla}\,P/nm \quad (3)$$

$$\underline{\nabla} \times \underline{E} = -(1/c)\,(\partial/\partial t)\underline{B} \quad (4)$$

$$\underline{\nabla} \,.\, \underline{E} = 4\pi\rho \quad (5)$$

$$\underline{\nabla} \times \underline{B} = (1/c)(\partial/\partial t)\underline{E} + (4\pi/c)\,\underline{j} \quad (6)$$

$$\underline{\nabla} \,.\, \underline{B} = 0 \quad (7)$$

This system of equations is highly nonlinear and can be linearized using perturbation techniques. In our case, let:

$$E(r,t) = E_o(r,t) + E_1(r,t) \quad (8)$$

$$B(r.t) = B_o(r,t) + B_1(r,t) \quad (9)$$

$$U(r,t) = V_o(r.t) + u_1(r,t) \quad (10)$$

$$n = n_o + n_1 \quad (11)$$

where E_o and B_o are the vacuum amplitudes for the laser pump wave and n_o is the equilibrium electron density (in the absence of all wave fields), v_o is the quiver velocity (the velocity of oscillation of an electron in the electromagnetic field). The subscript 1 refers to a perturbed quantity such that;
$|E_1| \ll |E_o|$, $|B_1| \ll |B_o|$, $|u_1| \ll |u_o|$ and $n_1 \ll n_o$

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In a homogeneous plasma of a constant density n_o, the laser wave will consider propagating with a large amplitude plane polarized. In general, to zero-order this wave may be represented as follows;

$E_o(r,t) = 2E_o \cos(k_o.r - \omega_o t)$ (12)

$B_o(r,t) = 2B_o \cos(k_o.r - \omega_o t)$ (13)

$V_o(r,t) = 2 V_o \sin(k_o.r - \omega_o t)$ (14)

Here, ω_o and k_o are the frequency and wave vector of the pump wave respectively, satisfying the dispersion relation;

where: $\omega_p = (4\pi n_o e^2/m)^{1/2}$ is the plasma frequency and c is the speed of light

$\underline{v}_o = e\underline{E}_o / m\dot{\omega}_o$ is the quiver velocity in the pump field

The first- order equations are:

$(\partial/\partial t)n_1 + n_o \underline{\nabla} . u_1 = 0$ (15)

$(\partial/\partial t) \underline{u}_1 = -(e/m)\underline{E}_1 - \underline{\nabla} (\underline{v}_o . \underline{u}_1) - 3v^2_o \underline{\nabla} n_1 / n_o$ (16)

$\underline{\nabla} \times \underline{E}_1 = -(1/c)(\partial/\partial t)\underline{B}_1$ (17)

$\underline{\nabla} \times \underline{B}_1 = (1/c)(\partial/\partial t)\underline{E}_1 - (4\pi e/c)(n_o u_1 + n_1 v_o)$ (18)

$\underline{\nabla} . \underline{E}_1 = -4\pi e\, n_1$ (19)

Using some normalization techniques to change all variables to dimensionless quantities such as considerable:

$\omega_o \longrightarrow 1$, $c \longrightarrow 1$, $e/m \longrightarrow 1$

$4\pi e^2 n/m\, \omega_o^2 \longrightarrow N$, let ; $3v_e^2/c = V_t$

Equations (15-19) can be reduced to the following system of second order differential equations:

$E'' - (p_2^2 + 2p_2 \gamma_p + w^2_{p0}(x)) E = -\omega^2_{po}(x)(u_o \upsilon_s)'$ (20)

$\upsilon_s'' - (p_1^2 + w^2_{p0} - \upsilon w^2_p(x)) \upsilon_s = \frac{P_1}{(P_1 + \upsilon)} u_o^* E'$

where : $P_1 = \gamma + i\,\omega_s$ for scattered light wave

And $P_2 = \gamma - i\,\omega$ for electron plasma wave

Using Fourier-Laplace transformation of the dependent variables, we get the dispersion relation

$(\omega_s^2 - \omega_p^2 - k_s^2 c^2)(\omega^2 - \omega_p^2 - 3k^2 v_e^2) = k^2 v_o^2 \omega_p^2$ (21)

This equation is a quadratic polynomial with real coefficients, represents the dispersion relation for Raman instability and it reflects the rule played by the laser driver in coupling the plasma electrostatic wave and the scattered electromagnetic wave in plasma. In the absence of v_o (no pump wave), the above relation recovers the dispersion relations for decoupled plasma and light waves. Finite laser amplitude couples these two modes giving rise to instability due to the resonant feedback loop.

3. Relativistic Effects

It has been shown [Lehmann and Spatschek (2011)] that the only nonlinear frequency shift as plasma waves get large is that due to the relativistic mass increase of the oscillating electrons. When $v_o \rightarrow c$ we find $m = m_o\beta$ and $\omega_p^2 = \omega_{po}^2/\beta$ where $\beta = (1 - v_o^2/c^2)^{-1/2}$ where ω_{po} refer to the rest mass and β is the relativistic factor corresponding to the mean electron velocity in the wave. Though other nonlinear effects can distort the wave, the fundamental frequency is unchanged because the electric force felt by each electron depends only on the number of charges on each side of it, And not on their positions. Laser accelerators benefit from the fortunate synchronism between the group velocity υ g of the laser pulse and the phase velocity the plasma waves. Since $\omega_p = \Delta\omega$ and $k_p = \Delta k$ **,** we have $v_\varphi = \omega_p / k_p = \nabla\omega / \nabla k = v_g$

Furthermore, since υ **g** is given by

$v_g = (1 - n/n_c)^{1/2} c$ where $n/n_c = \omega_p^2 / \omega_o^2$

And $v_g = v_\varphi$, the β- factor associated with v_φ is given by

$$\beta_\phi = (1 - v_\varphi^2 / c^2)^{-1/2} = [1 - (1 - n/n_c)]^{-1/2} = (n/n_c)^{-1/2} = \omega_o / \omega_p$$

The equations describing the behavior of the electrons are the relativistic fluids equations and Poisson's equation.

$$\partial n/\partial t + (1/\beta)\, \underline{\nabla} . (n\, \underline{u}) = 0 \quad (22)$$

$$(1/\beta)\, \partial\underline{u}/\partial t + (e/m\beta)\underline{E} - 3(v_e^2/\beta^2)(\underline{\nabla} n / n_o) = \underline{\nabla}\, (v^2/2\beta^2) \quad (23)$$

$$\underline{\nabla} \times \underline{E} = -(1/c)\, (\partial/\partial t)\underline{B} \quad (24)$$

$$\underline{\nabla} . \underline{E} = 4\pi e\, (n_o(x) - n) \quad (25)$$

$$\underline{\nabla} \times \underline{B} = (1/c)(\partial/\partial t)\underline{E} + (4\pi e/c\beta) nv \quad (26)$$

<u>This system of equations can be also reduced to give:</u>

$$(1/\beta)\, E'' - \left(p_2^2 + 2p_2\gamma_p + \frac{w^2{}_{p0}(x)}{\beta^3}\right) E/\beta = -(1/\beta^3)\, w^2{}_{po}(x)(u_o\, \upsilon_s)'$$

$$(1/\beta)\, \upsilon_s'' - \left(p_1^2 + \frac{w^2{}_{p0}}{\beta} - \frac{\upsilon w^2{}_p(x)}{\beta(p_1 + \upsilon)}\right) \upsilon_s / \beta = \frac{P_1}{(P_1 + \upsilon)}\, u_o^* E' / \beta^2) \quad (27)$$

And this can give the dispersion relation of the system if relativistic effects will be included.

$$\left(tk^2 + p_2^2 + 2p_2\gamma_p + \frac{w_{po}^2}{\beta}\right)\left(k_s^2 + p_1^2 + \frac{w_{po}^2}{\beta} - \frac{\upsilon\, w_{po}^2}{\beta(p_1 + \upsilon)}\right) = \frac{p_1}{(p_1 + v)} k^2 v_o^2 \omega_{po}^2 (1/\beta^3) \quad (28)$$

4. Numerical Calculations and Discussions

The stimulated Raman scattering system of boundary value differential equations with variable coefficients without including the relativistic effects, (20) have been solved numerically in an inhomogeneous plasma slap, $0 \le x \le \ell$, and a plasma channel immersed in this plasma with the boundary matrices which can be accounted for using the solutions of the dispersion relation (21) at both boundaries. The boundary conditions are chosen such that matching takes place in the homogeneous plasma channel region [Boyd and Barr (1988)]. However, the interaction region is assumed infinite through only resonant locally within the region

$0 \le x \le \ell$. The solution is developed to include the relativistic factor in equations (27) and (28). In figure (1), consider switching off the pump i.e. v_o= 0 or v_o< 0.002, which is the instability threshold (≈ 10^{11} W/cm^2). For this case the Airy function shape manifest itself and there is no plasma wave coupled to the injected light wave. This is expected for the case in which there is no pump and the wave number of the light wave is varies as a function of the density. This variation implies a variation of the group velocity of the wave. The group velocity decreases as the wave propagated towards the higher density region, where the energy must be conserved. Thus, the divergence of the energy flux must be zero. As the wave become closer to its reflection point, the wave energy density becomes higher; in the

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other word, the amplitude of the wave becomes higher to compensate for the slowdown of the group velocity of the wave near its turning point.

If we switch on the laser driver, $v_o \neq 0$ and higher than the instability threshold the two modes the light and plasma wave are coupled. Trying to avoid the negative relativistic effect, the value v_o is changed and the corresponding amplitudes for both amplified wave and plasma wave are represented in Figure (2). Figure (2) represents the relation between amplitude of the waves and the laser pump power in case of including the relativistic effect (Fig. 2-a) and without including relativistic effect (Fig. 2-b). The seed light wave is amplifying as it travels through the plasma. The group velocity of plasma wave (vg2) is much lower than that of the light wave (vg1) which mean that the plasma wave spends more time in the region of interaction than the light wave and the amplitude of the plasma wave is higher in the case of without relativistic effect than that with relativistic effect. Figures (2) shows that for the normalized electron velocity in the laser electromagnetic field $v_o/c = a_o \leq 0.06$, the relativistic effect can be neglected. In Figures (3), the value of $a_o = 0.06$ is used to illustrate the amplification process in a preformed plasma channel which started with the plasma slab and ended half way. The rest of the plasma slab will be continued with inhomogeneous over dense plasma. The density of the plasma slab was supposed to change linearly, and the preformed plasma channel can be moved inside this density. The starting and ending values of the channel on x-axis is varied to study the effect of no resonant plasma densities in front of the channel upon the seed wave amplification. The most profound amplification happens when the preformed resonant plasma channel started at the entrance of the laser pump pulse as shown in Figures (3). This high value of amplification was achieved by extending the resonance matching domain by the near homogeneity of the plasma density of the channel and by limiting the laser power to be as low as possible to exclude the undesired relativistic effects on amplification.

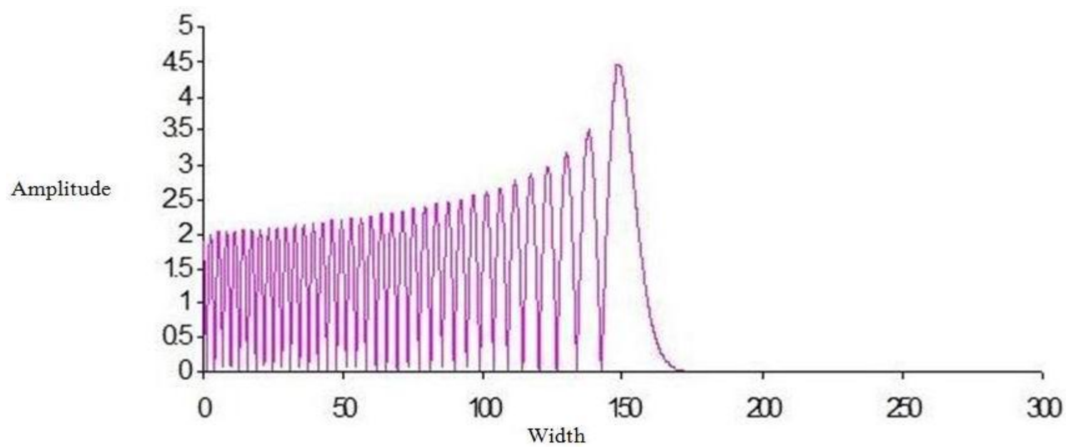

Fig. 1. Amplitude (A) versus the width (L) of the plasma for propagated light wave in an inhomogeneous plasma with $v_o = 0$.

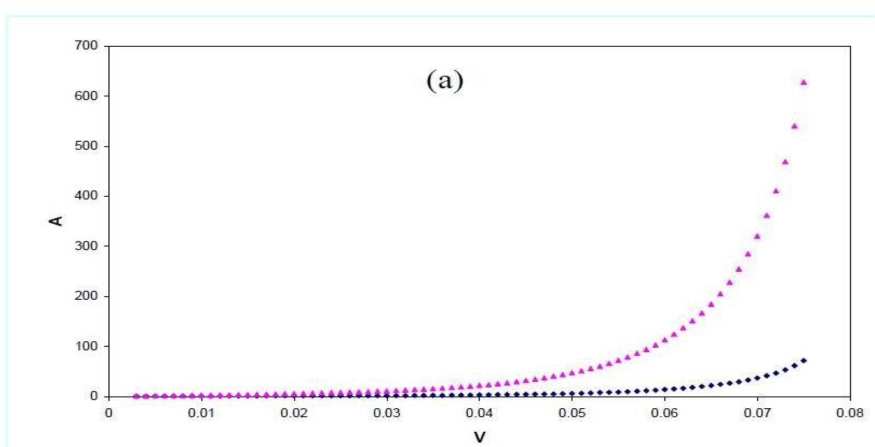

Fig. .2-a. Amplitude (A) versus velocity (v) for plasma (triangles) and scattered wave (squares) without relativistic effect included.

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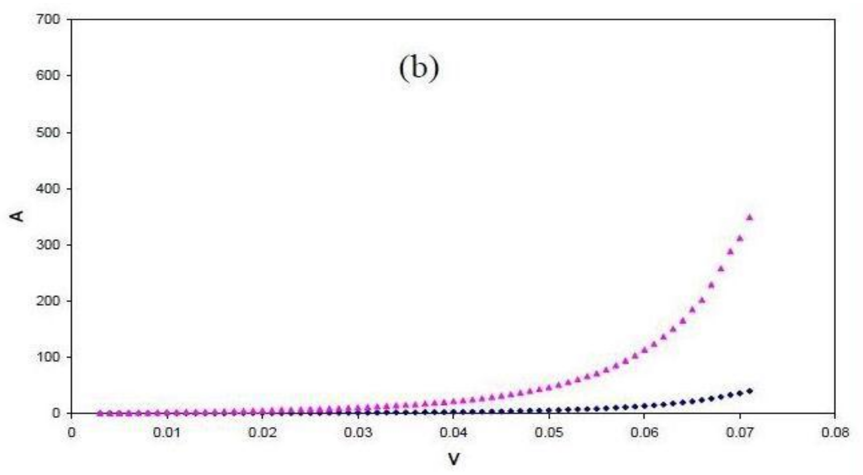

Fig. 2-b. Amplitude (A) versus velocity (v) for plasma (triangles) and scattered wave (squares) with relativistic effect included.

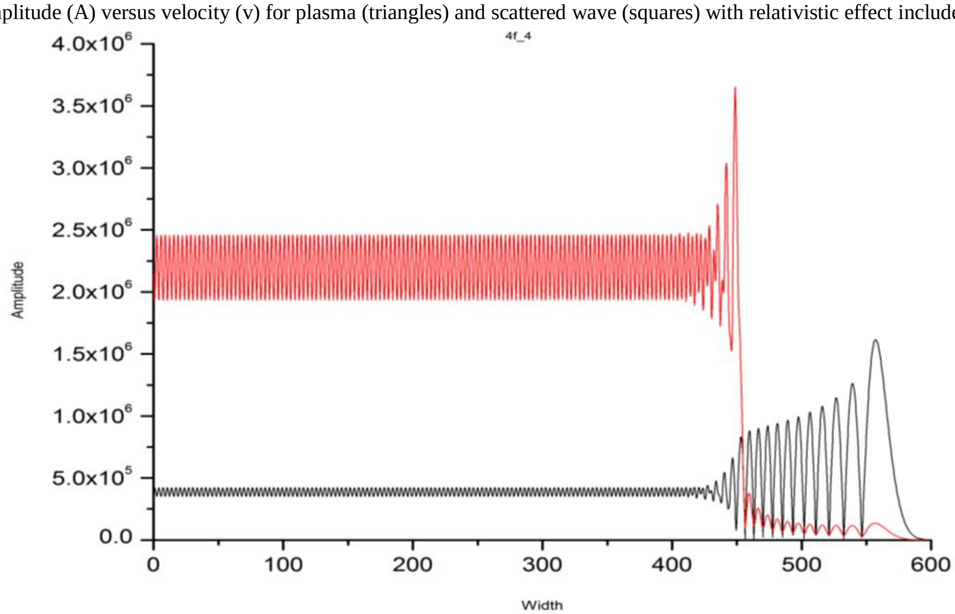

Fig. 3. Amplitude v_s. width of the plasma slab for the amplified seed electromagnetic wave (black) and the coupled plasma wave (red). The preformed plasma channel extends to 2/3 of the plasma slab. The remaining third is inhomogeneous over dense plasma including the resonance and reflection points of both waves. The inhomogeneous density part extends from $n_p / n_c = 0.15 – 0.44$.

5. CONCLUSION

The full wave equation of stimulated Raman scattering instability is solved in both homogeneous density plasma channel and inhomogeneous plasma slab. Numerical calculations are obtained by using a computer code and a good agreement is shown with analysis. The numerical model of stimulated Raman scattering equations is produced where the solution is given in a linear plasma density ramp.

These solutions indicate that laser amplification using the Raman backscatter interaction in a plasma channel appears very promising as a mean to achieve a high intensity pulse with a moderate and cheap total power in the pump. Such a system would enable the construction of nearly a meter scale experiments with peak intensities far above what is

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currently achieved with CPA amplifiers. With pulse amplification occurring over a sufficiently long interaction length, it is possible that a low power input laser could be used without reducing the peak power of the amplified laser pulse.

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