

# Projectile fragmentation of ${}^6,{}^7\text{Li}$ nuclei in photoemulsion at Dubna energy

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**Abstract-** Stripping and diffraction dissociation processes of  ${}^6\text{Li}$  at incident momentum 4.5AGeV/c and  ${}^7\text{Li}$  at incident momentum 3AGeV/c, provided by the JINR Synchrophastron, were studied using the photoemulsion technique. The two stable isotopes of  ${}^6\text{Li}$  and  ${}^7\text{Li}$  were treated as two weakly-bound systems composed of ( ${}^4\text{He}+d$ ) and ( ${}^4\text{He}+t$ ) cluster configurations, respectively. Along the track double scanning method was carried out in order to search for  ${}^6,{}^7\text{Li}$  interactions in emulsion (Em). For  ${}^6\text{Li}$ -Em about 1050 inclusive events were recorded with mean free path  $14.5 \pm 0.5$  cm while about 1015 events were detected for  ${}^7\text{Li}$ -Em with mean free path  $15.1 \pm 0.6$  cm. The type of momentum distribution inside the projectile according to which the features of each mechanism can be correctly explained and the difference between the angular distributions of these processes are theoretically discussed. The total cross sections of both stripping and dissociation are estimated theoretically as well as experimentally. The measured angular distributions of the outgoing fragments were compared with that calculated theoretically and good agreement between the experimental and theoretical values is obtained. This agreement supported the presence of  ${}^6,{}^7\text{Li}$  nuclei in a cluster mode at high energy reactions.

**Keywords-** Binding energy; Energy density function; Density depression.

## 1. INTRODUCTION

An extensive amount of experimental data on high energy nucleus-nucleus collisions has already accumulated since the last few decades. These data provide us with tremendous amount of information and motivated a large number of physicists to cover a wide range of aspects in nuclear physics. Some of these aspects are concerned with the study of reaction mechanisms and multiparticle production. For deeper understanding of complex processes in high energy reactions, the reactions are generally classified into central (with small impact parameters  $0 \leq b \leq |R_t - R_p|$ ) and peripheral collisions (with large impact parameters  $0 \geq b \geq |R_t - R_p|$ ). The study of central collisions usually concerns the occurrence of new phase transitions such as Quark-gluon plasma [Tokarev *et al.* (2011), Brown *et al.* (1993), Brown *et al.*(1991) and Ghosh *et al.* (1989)]and searching for new particles.

Projectile fragmentation processes, on the other hand, which form a category of peripheral interactions, are often believed to be more efficient in improving the understanding the various mechanisms that contribute in the continuum projectile spectra. One of the most important class of projectile fragmentation processes is the fragmentation of loosely-bound cluster-type projectile nuclei, such as  ${}^6\text{Li}$  and  ${}^7\text{Li}$ . The structure of  ${}^6\text{Li}$  and  ${}^7\text{Li}$  nuclei at low energies both in the theoretical treatments and in many experiments show a pronounced two-cluster configuration [Adamovich *et al.*(2004)]. Here  ${}^6\text{Li}$  and  ${}^7\text{Li}$  projectile nuclei are treated as two weakly- bound systems composed of ( ${}^4\text{He} + d$ ) and ( ${}^4\text{He} + t$ ) cluster configurations, respectively. In these nuclei an easily formed  $\alpha$ -cluster core leaves the other nucleons, less tightly connected with the core, free to form the other charged cluster with relatively high probability [Lepekhin *et al.* (1998)].

The projectile fragmentation processes of these nuclei are generally classified into:

- Stripping processes (inelastic breakup) [Serberand Robert. (1947) and Utsunomiya (1985)] in which one of the clusters misses the target and moves as if unaffected by the loss of its partner.
- Dissociation processes (elastic breakup) [Bertulaniet al. (1988), Peresadko *et al.* (2008; 2010), Sitenkoet al. (1985)] in which the projectile is elastically dissociates into its constituent clusters in the field of the target nucleus leaving the target in its ground state. In this process two mechanisms usually compete; the nuclear dissociation (diffractive dissociation) and the Coulomb dissociation (electromagnetic dissociation).

For studying the general features of fragmentation processes of complex projectiles, at high energies in the field

of the nucleus, the two stable isotopes  ${}^6\text{Li}$  and  ${}^7\text{Li}$  are used as useful interesting probes with the hope of showing the definite similarity in fragmentation features of both  ${}^6\text{Li}$  and  ${}^7\text{Li}$  nuclei.

This paper is devoted to the study of the most peripheral interactions of relativistic  ${}^{6,7}\text{Li}$  nuclei in nuclear photoemulsion (Em) at Dubna energy.

In section two we present the experimental work, while the theoretical treatment concerning stripping and dissociation fragmentation is given in section three. Section four is devoted to the results and discussions, followed by concluding remarks.

## 1. EXPERIMENTAL WORK

Two beams were used to irradiate emulsion stacks, the first is  ${}^6\text{Li}$  nuclei with momentum 4.5A GeV/c and the second is  ${}^7\text{Li}$  nuclei with momentum 3A GeV/c provided by the JINR Synchrophasotron. The emulsion layer were 550  $\mu\text{m}$  thick and of dimensions 10x20  $\text{cm}^2$ . The stacks were exposed in the beam parallel to the emulsion plane, so that the particles traversed the layers along its longer side. Along the track double scanning method was carried out in order to search for  ${}^{6,7}\text{Li}$  interactions in emulsion.

For  ${}^6\text{Li}$ - Em, about 1050 inclusive events were recorded with mean free path  $14.5 \pm 0.5$  cm while about 1015 events were detected for  ${}^7\text{Li}$ -Em with mean free path  $15.1 \pm 0.6$  cm. These results are in good agreement with the results of El-Nadi [El-Nadi *et al.* (1998; 2004)].

Out of these 1050  ${}^6\text{Li}$ - Em reactions, 779 elastic and inelastic projectile fragmentation reactions were found. While in  ${}^7\text{Li}$ -Em we collected 529 events having projectile fragments. All these events are accompanied by one or two non-interacting projectile fragments emitted with space angle  $\theta \leq 4^\circ$  with respect to the direction of incidence. These non-interacting fragments were identified in accordance with emulsion terminology as follows [Adamovich *et al.* (1977)]:

- Fragments with relative ionization ( $I / I_0 \leq 1.4$ ), where  $I_0$  is the plateau grain density, and not changing over a length are considered here as a singly charged ( $Z = 1$ ) projectile fragment. These may include a small admixture of pions which do not exceeds 10%. This background of pions had been separated easily by detecting the change in the ionization along a distance of 2 cm and hence it will not affect the present results.
- Fragments with relative ionization ( $I / I_0 \cong 1.4$ ) that does not change in ionization over a length of at least 2 cm from the point of interaction are considered as doubly charged ( $Z = 2$ ) projectile fragments.

As a matter of fact, using emulsion technique, the separation between the different types of projectile fragmentation processes is easily carried out as follows:

1. Stripping reactions: If one of the two clusters of the projectile nucleus will interact with the target while the other behaves as a spectator. It follows at once that we have one fragment (cluster) in the forward direction (with  $\theta \leq 4^\circ$ ). Under this criterion we observe either one singly charged ( $Z = 1$ ) or one doubly charged ( $Z = 2$ ) fragment in the forward direction. These events may or may not be accompanied by a target fragmentation. If no target fragmentations are observed, such events are the most peripheral ones and are known as "white stars". Such name arises from the emulsion terminology where the target fragmentations are called "heavily ionizing" particles. They are composed of both "black" and "grey" particles. Such white stars are therefore void of black and grey target fragmentations.
2. Dissociation reactions: According to dissociation mechanism, the projectile nucleus will dissociate into its constituent clusters by the Coulomb and/or nuclear field of the target without penetrating it and as a result of this we have two fragments in the forward direction (with  $\theta \leq 4^\circ$ ); one singly and one doubly charged clusters without detecting any target fragmentations, i.e., such events are all white stars.

## 2. THEORETICAL ANALYSIS

Now, the stripping and dissociation processes will be discussed theoretically in some detail.

### 2.1. Stripping Processes

In 1947 Serber [Serber and Robert (1947)], suggested the mechanism of stripping of deuteron according to which only one nucleon of a deuteron flying close to the target nucleus collides with it, while the other passes without interacting at all. Serber calculated both angular and energy distributions of the stripped particle by using two different assumptions; One which assumes the target is completely transparent to the outgoing particle (The transparent version)

and other which assumes the target is completely opaque to the outgoing particle (the opaque version). In 1982 Matsuse [Matsuse *et al.* (1982)], introduced the concept of a critical distance  $R_C$  between the two colliding nuclei. Utsunommiya [Utsunommiya (1985)] made an extension for the opaque version of Serber model to apply it to the case of heavy ion stripping reactions by incorporating the critical distance between constituent clusters in the projectile.

In the following we incorporate the critical distance concept in the transparent version of Serber and slightly modified the derivation in order to be compatible with the calculations of heavy ion reactions at high energies. The present model gives results similar to that of Sitenko [Sitenko *et al.* (1967)].

### 2.1.1. Geometrical calculation of stripping cross section

The stripping cross section can be easily estimated geometrically if we assume that this process takes place when the target nucleus covers not more than half the projectile. Then we have only to ask for the probability that at the instant of collision one of the two clusters of the projectile lie within a circle in a plane, perpendicular to projectile motion, of radius equal to the nuclear radius, while the other cluster will be outside it. We determine the stripping cross section  $\sigma_{st}$  by using the formula:

$$\sigma_{st} = \pi(R_T + \frac{R_P}{2})^2 - \pi R_T^2 \approx \pi R_T R_P$$

Where  $R_T$  and  $R_P$  are the radii of the target and projectile respectively, ( $R_T \ll R_P$ ). Taking into account the nuclear surface diffuseness,  $d$  [Berezhnoy *et al.* (1993)]. The stripping cross section was calculated by:

$$\sigma_{st} = \pi R_T R_P + (10/3)R_T d \tag{1}$$

The equation shows that the cross section of the stripping process increases with increasing the radius and/or the mass number of the target.

### 2.1.2. Angular distribution of fragments

Consider a stripping reaction which takes place between the projectile ( ${}^6\text{Li}$  or  ${}^7\text{Li}$ ) and one of the emulsion nuclei as a target. In such process one of the two clusters of the projectile will interact with the target nucleus, being stripped off, while the other will continue in its direction and emerge as a fragment. The momentum of the emitted cluster (projectile fragment)  $\mathbf{P}_f$  (Lab. momentum) can be determined as follows:

$$\mathbf{P}_f = \frac{A_f}{A_p} \mathbf{P}_p + \mathbf{P} \tag{2}$$

Where  $\mathbf{P}_p$  is the momentum due to the motion of the cm of the projectile and  $\mathbf{P}$  is the intrinsic momentum (i.e. the momentum due to the internal motion of the fragment inside the projectile) at the instant of interaction.

The probability density for finding the fragment with intrinsic momentum  $\mathbf{P}$  is proportional to the intrinsic momentum distribution of the emitted cluster inside the projectile;

$$P(\mathbf{p}) \propto |\phi(\mathbf{p})|^2$$

We use the Yukawa form for the space wave function of the relative motion:

$$\psi(\mathbf{r}) = C \left( \frac{\alpha}{2\pi} \right) \frac{e^{-\alpha r}}{r}$$

Where  $\alpha = \sqrt{2\mu\varepsilon} / \hbar$ ;  $\mu$  is the reduced mass,  $\varepsilon$  is the separation energy, i.e., the energy which must be imparted to the nucleus in order to separate the fragment, and  $C$  is a normalization constant. Such a wave function is an appropriate solution only when the radius of the nuclear potential well is smaller than the radius of the nucleus, thus this form of wave function is suitable for the loosely bound systems such as  ${}^6,7\text{Li}$  nuclei. In such a case the two clusters in the projectile nucleus spend most of their time outside the range of their mutual interaction.

Now incorporating the concept of the critical distance [Matsuse *et al.* (1982)],  $R_c$  where the constituent clusters will lose their identities inside this distance. The intrinsic wave function  $\psi(\mathbf{r})$  of the projectile, using the sharp cut off approximation can be written in the form:

$$\psi(\mathbf{r}) \propto \begin{cases} \frac{e^{-\alpha r}}{r} & \text{if } r \geq R_c \\ 0 & \text{if } r < R_c \end{cases} \quad (3)$$

The previous form of the wave function means that its amplitude is quickly damped at  $r < R_c$  and thus the Fourier transform  $\phi(\mathbf{P})$  under this condition is given by:

$$\phi(\mathbf{P}) \propto \int_{R_c}^{\infty} \psi_{Li}(\mathbf{r}) e^{-(i/\hbar)\mathbf{P}\cdot\mathbf{r}} d\mathbf{r}$$

Therefore the intrinsic momentum distribution of the clusters in the projectile is thus given by:

$$P(\mathbf{P}) \propto \frac{e^{-2\alpha R_c}}{(\alpha^2 \hbar^2 + p^2)^2} \left[ \cos\left(\frac{p}{\hbar} R_c\right) + \frac{\alpha \hbar}{p} \sin\left(\frac{p}{\hbar} R_c\right) \right]^2 \quad (4)$$

In the case of the loosely bound nuclei for which the intrinsic momentum  $p$  is governed by the separation of energy, i.e.  $p = \sqrt{2\mu\varepsilon} = \alpha \hbar$  and  $\alpha \hbar / p = 1$ .

Also for  ${}^{6,7}\text{Li}$  we have:

$$(p/\hbar)R_c \cong 0.306R_c, \quad R_c = 1.9\text{fm for } {}^6\text{Li}$$

$$(p/\hbar)R_c \cong 0.449R_c, \quad R_c = 2.02\text{fm for } {}^7\text{Li}$$

Therefore the quantity between squared brackets in (4) is of order of unity. Hence for simplicity we can safely approximate (4) as:

$$P(\mathbf{p}) \propto \frac{e^{-2\alpha R_c}}{(\alpha^2 \hbar^2 + p^2)^2}$$

Now if  $z$  is the direction of incidence, then  $p^2 = p_{\perp}^2 + p_z^2$  where  $p_z$  denotes the component of the intrinsic momentum in the incident direction, and one can get the probability of a given value of the perpendicular component  $p_{\perp}$  as:

$$P(p_{\perp}) 2\pi p_{\perp} dp_{\perp} \propto \frac{e^{-2\alpha R_c}}{(\alpha^2 \hbar^2 + p_{\perp}^2)^{3/2}} 2\pi p_{\perp} dp_{\perp} \quad (5)$$

Expressing (5) in terms of the space angle  $\theta$  of the emergent fragment in the laboratory system;  $\theta = \frac{p_{\perp}}{p_o}$  with

$p_o = \frac{A_f}{A_p} p_p + p_z$  is the momentum of the emitted fragment parallel to the incident projectile direction, and thus we

have:

$$p(\theta) d\Omega \propto \frac{\xi}{(\theta_o^2 + \theta^2)^{3/2}} d\Omega \quad (6)$$

Where  $\xi = e^{-2\alpha R_c} / p_o$ ,  $\theta_o = \alpha \hbar / p_o$  and  $d\Omega = 2\pi\theta d\theta$  is the element of the solid angle.

## 2.2. Dissociation process

In a projectile dissociation process, the projectile dissociates into its constituents by the Coulomb and/or nuclear field of the target nucleus. If the target is left in its ground state, the process will be called elastic break-up. The basic assumption of these kind of reactions is that the nuclei no overlap occurs between the projectile and target during their interaction, i.e.,  $b > R_p + R_T$ .

The break up processes of the projectile ( ${}^6\text{Li}$  or  ${}^7\text{Li}$ ) in the Coulomb or nuclear fields are of great importance and interest for the study of the nucleus structures of these nuclei, since they can give precious information about these structures.

### 3.2.1. Coulomb dissociation

The passage of a projectile of mass number  $A_p$ , charge  $Z_p e$ , velocity  $v$  and impact parameter  $b$  (larger than the nuclear interaction radius) by a target nucleus (mass number  $A_T$  and charge  $Z_T e$ ), initially at rest, will predominantly cause a momentum change for charged constituents of the projectile nucleus. This momentum change  $\Delta p$  is larger in the perpendicular direction to the projectile motion [Jackson *et al.* (1975)] and is given by:

$$\Delta p = \frac{2z_p z_T e^2}{bv}$$

From this we can calculate the energy transferred to the projectile nucleus as a whole:

$$\Delta E_A = \frac{(\Delta p)^2}{2m_{A_p}} = \frac{2(z_p z_T e^2)^2}{A_p m_n b^2 v^2}$$

where  $m_n$  is the nucleon mass and  $m_{A_p}$  is the projectile mass.

In very fast interactions we can assume the protons to move almost freely and the total amount of energy transferred to all protons will thus be:

$$\Delta E_z = z_p \frac{(\Delta p / z_p)^2}{2m_n} = \frac{2(z_T e^2)^2 z_p}{m_n b^2 v^2}$$

The difference gives the internal excitation energy of the nucleus as:

$$\Delta E_{\text{int}} = \Delta E_z - \Delta E_A = \frac{2N_p z_p (z_T e^2)^2}{A_p m_n b^2 v^2} \quad (7)$$

where  $N_p = A_p - Z_p$

We must note that, the role of the target and projectile can be exchanged, i.e., we can consider the case of internal excitation of the target by the electromagnetic field of the projectile and vice versa.

The Coulomb contribution to the nuclear fragmentation in relativistic heavy ion collisions is a two-step process involving the excitation of the nucleus followed by the decay of this nucleus.

### 3.2.2. Cross section of Coulomb dissociation

According to Bertulani [Bertulani *et al.* (1988)], if we restrict ourselves to the electric dipole  $E_1$  and the electric quadrupole  $E_2$  modes which are the most important multipolarities for the projectile, one can get the Coulomb dissociation differential cross section as:

$$\frac{d\sigma_c}{dQ} = \frac{d\sigma_{E_1}}{dQ} + \frac{d\sigma_{E_2}}{dQ} \quad (8)$$

With:

$$\frac{d\sigma_{E_1}}{dQ} = 128Z_T^2 \eta^2 (c/v)^2 (\beta_1 z_1 - \beta_2 z_2)^2 \frac{\alpha Q^4}{(\alpha^2 + Q^2)^4} \times [\zeta K_0 K_1 - \frac{v^2 \zeta^2}{2c^2} (K_1^2 - K_0^2)]$$

$$\frac{d\sigma_{E_2}}{dQ} = (512/15)Z_T^2 \eta^2 (c/v)^4 (\beta_1 z_1 + \beta_2 z_2)^2 \frac{\alpha Q^6 (\omega/c)^2}{(\alpha^2 + Q^2)^6} \times [\frac{2}{\gamma^2} K_1^2 + (2 - v^2/c^2) \zeta K_0 K_1 - \frac{v^4 \zeta^2}{2c^4} (K_1^2 - K_0^2)]$$

Where  $Q$  is the relative motion momentum of the outgoing fragments,  $Z_T$  is the atomic number of the target nucleus,  $\eta$  is the fine structure constant,  $C$  is the speed of light,  $v$  is the projectile velocity,  $Z_1, Z_2$  are the atomic numbers of the two clusters,

$\alpha = \sqrt{2\mu\xi} / \hbar$ ,  $\omega$  is the angular frequency.  $\gamma = \sqrt{1/(1 - v^2/c^2)}$ ,  $\zeta = \omega R / \gamma v$  and  $K_0, K_1$  are the modified Bessel functions of order zero and one respectively.

### 3.2.2. Nuclear dissociation

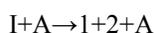
When a loosely bound projectile consisting of two clusters passes near enough to a target nucleus so that it is affected by the nuclear field of this target, there is a large probability that the projectile nucleus will dissociate diffractively into its two clusters. For energies sufficiently high, the particle interaction is known to be diffractive. Diffractive phenomena occur if the wavelength associated with the relative motion of colliding particles is small compared to the characteristic dimension of the interaction region. Hence the criteria for the diffraction are readily satisfied at high energies. Interactions between nuclei and composite particles are accompanied by various diffraction processes.

Therefore dissociation of the  ${}^6\text{Li}$  nuclei will play a very important role and will represent an important component in the total cross section.

#### 3.2.2.1. Cross section of nuclear dissociation

The phenomenon of the diffractive dissociation was theoretically predicted for the first time for the deuteron by Feinberg [Feinberg et al. (1955)], Glauber [Glauber et al. (1955)] and Akhiezer and Sitenko [Akhiezer and Sitenko (1957a; 1957b)] independently. Later on, Sitenko [Sitenko et al. (1967)] pointed out that the diffraction processes can also occur during the interaction of weakly-bound cluster-type light ions accelerated up to the comparatively high energies. In 1985 Sitenko [Sitenko et al. (1985)] illustrated by typical examples that the stripping and dissociation reactions of the loosely-bound cluster-type nuclei provide the main contribution to the inclusive spectra of the projectile fragments. In this section we present a brief account about the theoretical formalism of the diffraction dissociation for loosely-bound cluster-type nuclei which have the possibility to dissociate into two clusters.

Consider a break-up process of a relativistic ion  $I$ , consisting of two fragments (clusters) 1 and 2, on a target nucleus  $A$ . This process can be written in the symbolic form:



The total amplitude for dissociation  $f_d$  of a weakly-bound projectile incident on a target nucleus, assumed to stay in its ground state, contains contributions from diffractive dissociation on the target surface  $f_N$  and from coulomb dissociation  $f_C$  for impact parameter  $b$  larger than the sum of the nuclear interaction radii, i.e.,  $f_d = f_N + f_C$ .

Neglecting, for the time being, the coulomb interaction, the amplitude of the two-body diffraction dissociation of the incident ion on the target nucleus is given by Bertulani [Bertulani *et al.* (1988)]:

$$f_N(q, Q) = \frac{ik}{2\pi} \int db e^{iq \cdot b} \int dr \psi_f^*(r) \Gamma_d(b) \psi_i(r) \quad (9)$$

where  $k$  is the C. M. momentum of the projectile,  $q$  is the momentum change in the scattering ( $q = 2k \sin(\theta/2)$ ;  $\theta$  is the scattering angle),  $Q$  is the relative motion momentum of the outgoing fragments,  $\Gamma_d$  is the profile function for the diffractive dissociation,  $b$  is the projection of the CM radius vector of the incident ion in the plane normal to the incident beam, i.e., the impact parameter of center of mass of the projectile and  $\psi_i$  &  $\psi_f$  are the wave functions of the projectile ground state and the final state of the relative motion of the outgoing fragments which are emitted as a result of ion dissociation, respectively.

We must realize that, the incident ion during the diffraction dissociation passes from the bound state to continue spectrum of unbound state and as a result of this the amplitude must contain the two previous wave functions. These functions from a complete system of orthonormality functions satisfying the following relation:

$$\psi_i(r) \psi_i^*(r') + \frac{1}{(2\pi)^3} \int \psi_f^*(r) \psi_f(r') d^3q = \delta(r - r') \quad (10)$$

The total profile function  $\Gamma_d(b)$  is directly determined by the profile functions for respective clusters  $\Gamma_1(b_1)$  and  $\Gamma_2(b_2)$  with the help of the relation:

$$\Gamma_d(b) = \Gamma_1(b_1) + \Gamma_2(b_2) - \Gamma_1(b_1) \Gamma_2(b_2)$$

where  $b_1$  and  $b_2$  are the projections of the cluster radius vectors, respectively. The functions  $\Gamma_1(b_1)$  and  $\Gamma_2(b_2)$  characterize the strong absorption of clusters 1 and 2 by the target and are related to the amplitudes of cluster-nucleus scattering through the two-dimensional Fourier transformations

$$\Gamma_i(b) = \frac{1}{2\pi i k} \int dq_i e^{iq_i \cdot b} f_i(q_i), \quad i = 1, 2;$$

$$f_i(q_i) = iKR \frac{J_1(q_i R)}{q_i}, \quad q_i = \beta_i q$$

where  $\beta_1 = m_2 / (m_1 + m_2)$ ,  $\beta_2 = m_1 / (m_1 + m_2)$ ,  $R = 1.2 A^{1/3} \text{fm}$  is the radius of the target nucleus and  $J_1$  is the Bessel function of order one.

In the case of very loosely-bound projectile nuclei we can describe the relative motion of the clusters within the projectile and after dissociating by the wave functions:

$$\psi_i(r) = \sqrt{\frac{\alpha}{2\pi}} \frac{e^{-\alpha r}}{r}$$

$$\psi_f(r) = \exp(iq \cdot r) + \frac{1}{iq - \alpha} \frac{e^{-qr}}{r}$$

respectively

These wave functions correspond to the assumption of zero-range nuclear forces between clusters in the projectile. The two wave functions satisfy the orthonormal relation eq. (10) and thus we can write:

$$f_N(q, Q) = iKR \left\{ \frac{J_1(qR)}{q} [F(-\beta_1 q, Q) + F(\beta_2 q, Q)] - \frac{ikR}{2\pi} \int d^2q' \frac{J_1(q'R)}{q'} \frac{J_1(|q - q'|R)}{|q - q'|} F(-\beta_1 |q - q'|, Q) \right\} \quad (11)$$

with:

$$F(q, Q) = \int \psi_f^*(r) e^{i\mathbf{q}\cdot\mathbf{r}} \psi_i(r) d^3 r$$

$$= (8\pi\alpha)^{1/2} \left\{ \frac{1}{\alpha^2 + (q-Q)^2} + \frac{1}{2q(i\alpha-Q)} \ln \left( \frac{Q+q+i\alpha}{Q-q+i\alpha} \right) \right\}$$

The first term in eq. (11) describes the independent scattering of separate clusters by the nucleus (in the impulse approximation) and the second one corresponds to the simultaneous scattering, of the clusters, the so-called eclipse term.

Within the framework of the diffraction model [Sitenko *et al.* (1967)], the differential cross section of the ion dissociation on nuclei is given by:

$$d\sigma_N = |f_N(q, Q)|^2 d\Omega \frac{dQ}{(2\pi)^3}, d\Omega = \frac{2\pi}{k^2} q dq \quad (12)$$

Inserting eq. (11) into eq. (12), using the orthonormality conditions of the wave functions, the integration over Q can be easily performed and one gets:

$$\frac{d\sigma_N}{dq} = \frac{2\pi R}{q} J_1^2(qR) \left\{ 1 + \frac{2\alpha}{q} \arctan\left(\frac{q}{2\alpha}\right) - \frac{2\alpha^2}{q^2} \left( \frac{1}{\beta_1} \arctan\left(\frac{\beta_1 q}{2\alpha}\right) + \frac{1}{\beta_2} \arctan\left(\frac{\beta_2 q}{2\alpha}\right) \right)^2 \right\} \quad (13)$$

To obtain the total diffractive dissociation cross section, one has to integrate eq. (13) numerically. For small scattering angles (i.e., at high energy), the diffraction dissociation amplitude can be calculated in the impulse approximation. In this case the second term in the last equation is disregarded, where the contribution of double scattering is insignificant.

### 3.2.2.1. Angular distribution of the outgoing fragments

For simplicity and without loss of generality, we shall take the wave functions of the cluster relative motion before and after collision to be [Sitenko *et al.* (1985)]:

$$\psi_i(r) = (2\xi^2/\pi)^{3/4} \exp(-\xi^2 r^2)$$

$$\psi_f(r) = \exp(iq \cdot r) - \sqrt{8} \exp\left(\frac{-q^2}{4\xi^2 - \xi^2} - \xi^2 r^2\right)$$

where  $\xi$  is related to the r.m.s radius by:

$$\xi = \frac{1}{(2/\sqrt{3})\langle r^2 \rangle^{1/2}}$$

To obtain the angular distribution of the second fragment, it is necessary to pass eq. (12) from the variable Q to  $k_{2z}$  and  $q_2$  which are the components of the cluster 2 wave vector parallel and perpendicular to k, respectively.

Integrating eq. (12) over the variable q and the directions of the two-dimensional vector  $q_2$ , we obtain the energy distribution of the second fragment in the solid angle  $d\Omega_2$  as:

$$\frac{d^2\sigma}{d\Omega_2 dE_2} = \frac{(kR)^2 \beta_2^2}{2\gamma^2 \sqrt{\pi L E_2}} \exp\left(\frac{(\sqrt{E_2} - \sqrt{\beta_2 E})}{L} - \frac{x_2^2}{p^2}\right) x \int_0^\infty dx x g^2(x) \sum_{i=1}^6 H_i(x, x_2) \quad (14)$$

where:

$$g(q) = R \frac{J_1(qR)}{q}$$

$x = qR, x_2 = q_2 R, p = \sqrt{2\xi}R, L = \hbar^2 \xi^2 / m_2, E$  is the incident particle energy, and the functions  $H_i(x, x_2)$  are of the form:

$$H_1(x, x_2) = 1,$$

$$H_2(x, x_2) = \left\{ \exp\left(-\frac{3\beta_2^2}{4p^2} x^2\right) + \exp\left(-\frac{2\beta_2^2 + \beta_1^2}{4p^2} x^2\right) \right\} I_0\left(\frac{2\beta_2}{p^2} x x_2\right),$$

$$H_3(x, x_2) = 2 \left\{ \exp\left(-\frac{3\beta_2^2}{p^2} x^2\right) + \exp\left(-\frac{2\beta_2^2 + \beta_1^2}{4p^2} x^2\right) \right\} I_0\left(\frac{\beta_2}{p^2} x x_2\right),$$

$$H_4(x, x_2) = 2 \exp\left(-\frac{1}{2p^2} x^2\right) I_0\left(\frac{1}{p^2} x x_2\right),$$

$$H_5(x, x_2) = -2 \left\{ \exp\left(-\frac{3\beta_2^2 + 2}{4p^2} x^2\right) + \exp\left(-\frac{2\beta_2^2 + \beta_1^2 + 2}{4p^2} x^2\right) \right\} I_0\left(\frac{\beta_2 + 1}{p^2} x x_2\right),$$

$$H_6(x, x_2) = \exp\left(-\frac{1}{p^2} x^2\right) I_0\left(\frac{2}{p^2} x x_2\right)$$

Integrating eq. (14) over energy, we obtain the angular distribution of the outgoing fragment;

$$\frac{d\sigma}{d\Omega_2} = \frac{(kR)^2 \beta_2^2}{\gamma^2} \exp\left(-\frac{x_2^2}{p^2}\right) \int_0^\infty dx x g^2(x) \sum_{i=1}^6 H_i(x, x_2) \quad (15)$$

The most probable angle of the emergent fragment can be evaluated simply from The momentum change  $\Delta p = (2\mu\varepsilon)^{1/2}$  due to decay ,which is usually perpendicular to the beam, and the incident momentum  $p_o$  as:

$$\tan(\theta) = \Delta p / p_o$$

### 3. RESULTS AND DISCUSSIONS

In the following we present the results of measurements for the projectile fragmentation of  ${}^6\text{Li}$  and  ${}^7\text{Li}$  nuclei in Em. A comparison between the measured and theoretical cross sections and angular distributions for the fragments produced in stripping and dissociation projectile fragmentation reactions are displayed. Table 1 shows the details of the scanned measurements for  ${}^6\text{Li}$ -Em and  ${}^7\text{Li}$ -Em. It also contains a comparison between the experimental and theoretical calculations of dissociation and stripping cross sections.

**Table 1.** The scanned measurement details of  ${}^6\text{Li}$  and  ${}^7\text{Li}$  beams. A comparison between experimental and theoretical values of both dissociation and stripping cross sections are displayed.

Projectile	Energy A Gev	Total Number Of Events	Total Scanned Length (m)	Mean Free Path (cm)	Cross Section Of Dissociation (mb)		Cross Section Of Stripping (mb)	
					exp.	theor.	exp.	theor.
${}^6\text{Li}$	3.7	1021	148.46	14.54±0.5	75±8.20	65	367.33±18.51	374
${}^7\text{Li}$	2.2	1011	154.68	15.3±0.6	54.83±6.50	42.5	391.98±22.42	394

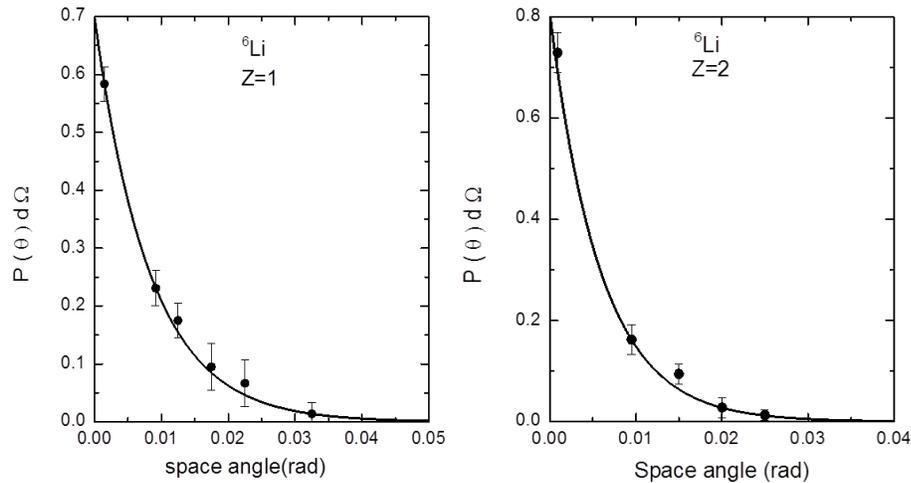
Comparing the values of the mean free paths with corresponding values of projectiles with mass number ranging from proton up to silicon shows out that the values obtained here deviate slightly from the well-established parameterizations [Bradt and Peters (1950)] obeyed by these projectiles. However, they agree well with the results printed in [El-Nadi et a. (1998; 2004)]. The experimental values of the stripping cross section are found to be in very good agreement with the calculated ones, according to eq. (1) taking into account the nuclear surface diffuseness.

For dissociation process, two mechanisms usually compete, namely, Coulomb (electromagnetic) and nuclear (diffractive) dissociation. Unfortunately, at high energy the separation between the two mechanisms by investigating the angular distributions of the products, which is possible at low and intermediate energies, become impossible. The calculations of the cross sections for the two processes, the Coulomb dissociation  $\sigma_c$ , through eq.(12) and the nuclear dissociation  $\sigma_N$ , through eq. (13) shows that there is a negligible probability for Coulomb dissociation to take place and the dissociation process is a pure diffractive one as shown in table 2.

Table 2. A comparison of the calculated nuclear and Coulomb cross sections for  ${}^6\text{Li}$  and  ${}^7\text{Li}$  beams

Projectile	Energy A Gev	$\sigma_N$	$\sigma_c$
${}^6\text{Li}$	3.7	65 mb	0.02 mb
${}^7\text{Li}$	2.2	42.5 mb	0.21 mb

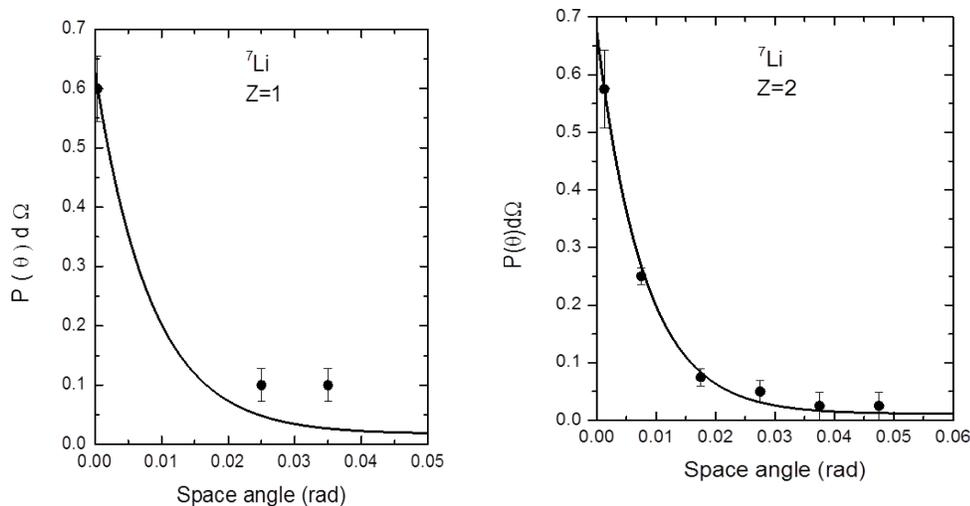
For more verification we estimated the excitation energy of  ${}^6\text{Li}$  nucleus results from its interaction with any nucleus inside the emulsion, by eq. (7) and we found that the maximum value of this energy is 0.2 MeV when  ${}^6\text{Li}$  interacts with the heavy nucleus in Em (Ag) while the first excited state of  ${}^6\text{Li}$  is at 2.19 MeV. Thus we deduce that there is no possibility for the Coulomb dissociation to take place at all and the dissociation is pure diffractive.



**Fig. 1.** Angular distribution of fragments produced from stripping reactions of  ${}^6\text{Li}$ -Em. Solid lines are the deuterons and  $\alpha$ -particles distributions as calculated according to eq.(6).

As a proposed assumption of  ${}^6\text{Li}$  structure we compare the angular distribution data with the deuteron and  ${}^4\text{He}$  angular distributions, calculated using eq. (6). These are displayed by the solid lines in fig.1.

Similarly, the triton and  ${}^4\text{He}$  angular distributions are compared with the data in fig. 2. for  ${}^7\text{Li}$  projectile beam. The



**Fig. 2.** Angular distribution of fragments produced from stripping reactions of  ${}^7\text{Li}$ -Em. Solid lines are the tritons and  $\alpha$ -particles distributions as calculated according to eq.(6).

observed agreement between theoretical and experimental results strongly supports the proposed structures for  ${}^6\text{Li}$  as  $(\alpha + d)$  and  ${}^7\text{Li}$  as  $(\alpha + t)$ . Events with stripping fragments  $Z=1$  for  ${}^7\text{Li}$ -Em were found to be much smaller than events having stripping fragments with  $Z=2$ , i.e., the triton component may have a most probable choice to participate in the reaction, while the  $\alpha$ -particle is mostly stripped out. The non-interacting projectile fragments (spectators) are confined in a forward cone of space angle  $\theta = 0.2 / P_{\text{beam}}$  where the value 0.2 is the average Fermi momentum and  $P_{\text{beam}}$  is the incident beam momentum per nucleon (in GeV). Accordingly for the present momenta all charged particles having

space angle  $\theta \leq 4^\circ$  and ionization equal to or less than that of the incident beam are considered as projectile fragments. An immediate consequence of this, is that the singly charged projectile fragments should emerge in a cone with space angle extending to  $3^\circ$ . Surprisingly, in  ${}^6,7\text{Li}$  interactions accompanied with only one singly or doubly charged projectile fragments, the space angle of the fragments is always  $\leq 1^\circ$  in figs. (1, 2). Thus the expected angular distribution is much wider than the measured one.

Fig. 3 and fig. 4 show the angular distributions of the singly (shower) and doubly (grey) fragments from  ${}^6\text{Li}$  and  ${}^7\text{Li}$  dissociation process, respectively. The calculated distributions, according to eq. (14) for  $\alpha + d$  in  ${}^6\text{Li}$ -Em and for  $\alpha + t$  in  ${}^7\text{Li}$ -Em reactions are presented by the solid lines.

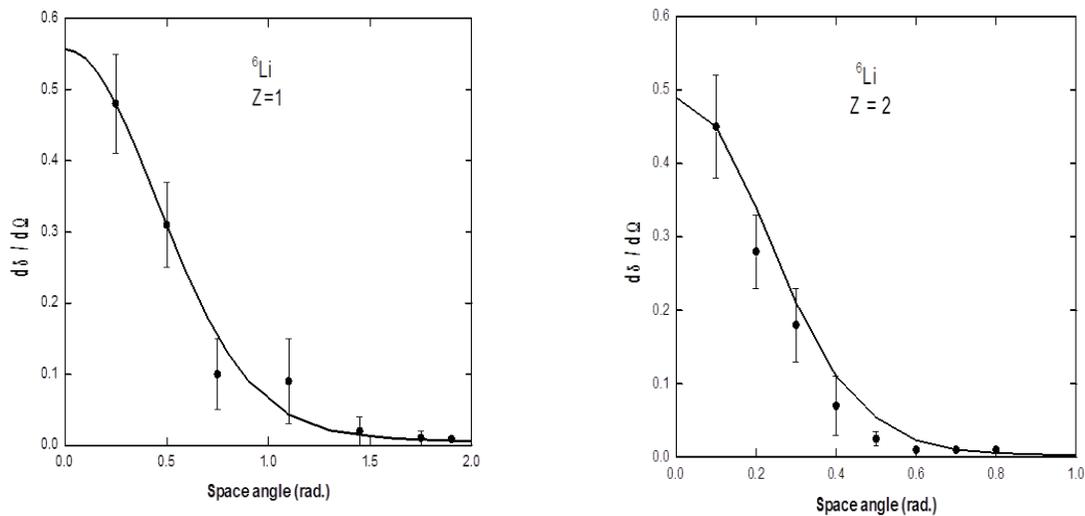


Fig. 3. Angular distribution of fragments produced from dissociation reactions of  ${}^6\text{Li}$ -Em. Solid lines are the calculations according to eq.(14).

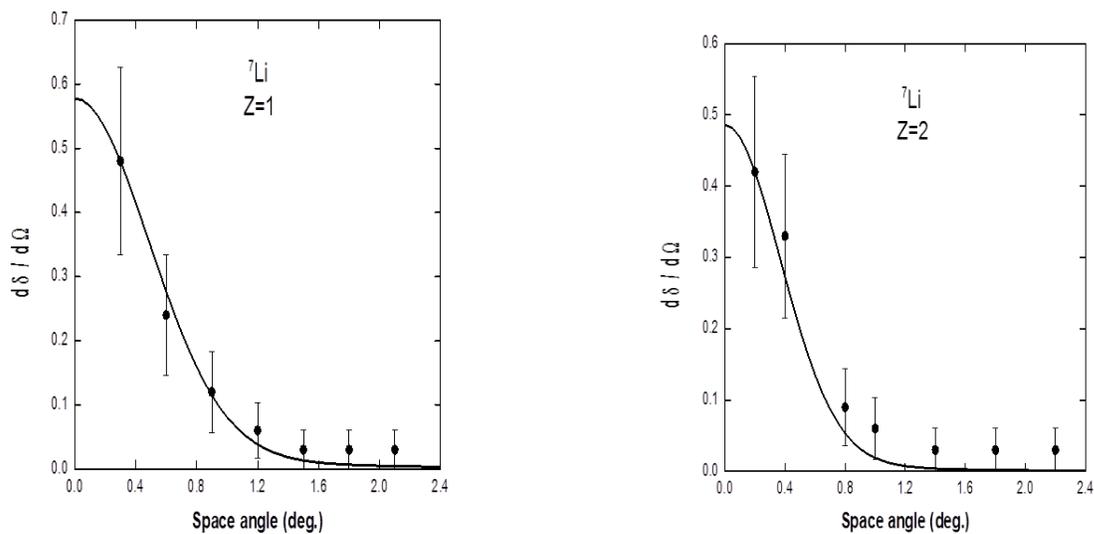


Fig. 4. Angular distribution of fragments produced from dissociation reactions of  ${}^7\text{Li}$ -Em. Solid lines are the calculations according to eq.(14).

Good agreement is clearly observed between the experimental results and calculations. The transparent version of Serber model which has been extended by incorporating the concept of the critical distance  $R_C$  between the two clusters with slight modification is found to be compatible with the experimental data. The estimated value of  $R_C$  is  $\approx 2$  fm for  ${}^6,7\text{Li}$  with the assumed structures which means that the two clusters are, in most of time, outside the range of their mutual interactions ( $\approx 1$  fm) and it follows, at once, that the quasi-free stripping of Serber can take place.

In conclusion it is shown that the characteristics of the stripping and dissociation reactions depend primarily on the fact that  ${}^6,7\text{Li}$  nuclei are very loosely bound systems consisting of two clusters with low binding energy, and that the two clusters actually spending most of their time outside the range of their mutual forces. Also the stripping and dissociation reactions represent the main components of the total cross section of  ${}^6,7\text{Li}$  inelastic interactions and have the main contribution in the particle spectra produced from  ${}^6,7\text{Li}$  fragmentation reactions.

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