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Results on the scaling of multiplicity distributions of fast target fragments in high energy nucleus-nucleus collisions

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Abstract- In this work the fast target fragments from high multiplicity interactions of 16O (at 60A GeV and 200A GeV) and 32S (at 3.7A and 200A GeV) ions with Ag (Br) targets have been measured. The characteristics of these interactions have been compared to those from simulations using the Modified FRITIOF Code. The comparison indicates that there is a need to modify the code and incorporate a greater amount of rescattering for a better fit to the experimental data. The multiplicity distributions for all interactions have been fitted well with the Gaussian distribution function. The measurements of the scaled variance (ω >1) show that the production of target fragments at high energies cannot be considered as a statistically independent process. The energy dependence of entropy is examined. The entropy values normalized to average multiplicity (S /< Ng >) are found to be energy independent. The possibility of scaling, i.e., similarity in the multiplicity distributions of grey tracks produced in nucleus-nucleus interactions has been examined. A simplified universal function has been used to display the experimental data. The relationship between the entropy, the average multiplicity and the KNO function is examined as well.

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1. INTRODUCTION

Study of the secondary charged particles produced in heavy ion collisions is attracting a great deal of attention during the recent years. Measurements of multiplicities yield some insight into the collisions themselves but are also crucial as input parameters for a multitude of models, explaining various phenomena at later stages in the collisions. The study of the global distribution, from the earlier well-known KNO scaling and its violation [1, 2] to a recently novel scaling form [3], can give us a lot about the dynamical features of the multiparticle production processes. It is reported that the multiplicity distribution (MD) of secondary charged particles produced in high energy hadron-hadron and hadron-nucleus collisions Obey Koba, Nielson and Olesen (KNO) scaling [1]. However, no attention has been paid to study the nature of the MD of secondary charged particles produced in relativistic heavy ions reactions. However, to unearth the dynamics of target fragments in the high Nh-events (mainly contributed by heavy target (AgBr) in emulsion, we study and analyze the MD of grey (g-) particles emitted in the interactions of 32S beams with AgBr at 3.7A and 200A GeV as well as the interactions of 16O with AgBr at 60A and 200A GeV. In his 1969 paper [4], Feynman concludes that the mean total number of particles rises logarithmically with √ s. He argues that the probability of finding a particle of type i, mass m, transverse momentum pt, and longitudinal momentum Pz is of the form:

$$P_i(P_T, P_Z, m) = f_i(P_T, P_Z/W)\frac{dP_Z d^2 p_T}{E} \tag{1}$$

where the energy of the particle E, and the parameter is given by:

$$E = \sqrt{m + {P_T}^2 + {P_Z}^2} = \sqrt{{m_T}^2 + {P_Z}^2} \quad \text{and} \quad W = \frac{\sqrt{s}}{2} \tag{2}$$

The function fi (pt; pz/W) is a structure function and is known as the Feynman function. Feynman's assumption is that fi is independent of W, which is called Feynman scaling.

Using the invariant cross section, σ the integration of Eq. (1) under the assumption Feynman made (W is large), can give the mean multiplicity in the form

$$< n > \propto \ln(W) \propto ln\sqrt{s} \tag{3}$$

The concept of Feynman scaling was the main assumption when Koba, Nielsen, and Olesen suggested a similar scaling in 1972. This scaling is now called KNO scaling [1].

One of the most influential contributions to the analysis of multiplicity distributions was made by KNO. They put forward the hypothesis that at very high energies the probability distributions Pn for detecting n final state particles exhibit a scaling law of the form

$$P(n) = \frac{1}{<n>}\Psi(Z) = \frac{\sigma_n}{\sigma_{inel}} \tag{4}$$

The variable $Z = n/<n>$ stands for normalized multiplicity, $<n>$ represents the average number of charged secondary particles, σ_n is a partial cross-section for producing n charged particles and σinel is the total inelastic cross-section.

That is to say, the $<n>P_n$ measured at different energies (i.e. $<n>$) scale to the universal curve Ψ when plotted against the multiplicity n rescaled by the average multiplicity $<n>$. The scaling function $\Psi(Z)$ must satisfy the normalization conditions:

$$\int_0^\infty \Psi(Z)dZ = \int_0^\infty Z\,\Psi(Z)dZ = 1 \qquad \text{(i.e. } <z>=1) \tag{5}$$

Obviously, the moments Mq of $\Psi(Z)$ are independent of collision energy if Eq. (4) is satisfied,

$$M_q = \frac{<n^q>}{<n>^q} \qquad \text{Where } q = 2, 3, 4 \ldots \tag{6}$$

Besides Ψ(z), there is a second properly normalized scaling function obeyed by the P_n. Hegyi [5] has demonstrated that in addition $<n>P_n$ the more simple combination nP_n also scales to a universal curve in the variable $n/<n>$ if KNO-scaling holds valid. This yields the scaling law for the MDs in the form

$$\varphi(Z) = \mathrm{n}\,P_n \tag{7}$$

The obvious advantages of this new scaling are (i) the nP_n are not influenced by the statistical and systematic uncertainties of $<n>$, hence $\varphi(Z)$ provides more selective power than the original KNO-scaling function Ψ(z) and (ii) the new scaling function generates scale parameter σ = 1 since it depends only on the combination of z and the scale parameter of Ψ(z).

2. EXPERIMENTAL TECHNIQUE

Two stacks of nuclear emulsions were horizontally exposed to 32S ion beams at two widely differing energies. The first stack of Br-2 emulsion pellicles was irradiated by 3.7A GeV at Dubna Synchrophasatron and the second one of FUJI films was exposed to 200A GeV at the CERN-SPS (Exp. no. EMU03). Additionally, two stacks of nuclear emulsions were horizontally exposed to 16O ion beams at the CERN- SPS. The first stack of the FUJI films was irradiated by 60A GeV and the second one of the ILFORD–G5 was exposed to 200A GeV. The chemical composition of the used emulsion types are shown in Table (1).

Table (1): The chemical composition of the used emulsion types (Atoms/cm^3 ×10^{22}).

Element	1H	12C	14N	16O	32S	80Br	108Ag	133I
FUJI	3.2093	1.3799	0.3154	0.9462	0.0134	1.0034	1.0093	0.0055
ILFORD-G5	3.1900	1.3900	0.3200	0.9400	0.0140	1.0100	1.0200	0.0060
NIKFI-BR2	3.1500	1.4100	0.3950	0.9560	-	1.0280	1.0280	-

The pellicles were scanned under 100× magnifications with an “along-the-track” scanning technique. Each beam track was carefully followed up to a distance of 5cm or until it interacts with an emulsion nucleus. Other details about irradiations and scanning are given in earlier publications [6-9]. At all energies, the total samples of inelastic interactions with emulsion nuclei were analyzed by studying the tracks emitted from each interaction detected. Depending on the commonly accepted emulsion experiment terminology [10], the tracks of secondary charged particles generated in each interaction were classified according to their ionization, range and velocity as:

(a) Black (b) particles are those having range of L < 3 mm, corresponding to protons with kinetic energy ≤ 26 MeV. They are mainly due to evaporated target fragments. Their multiplicity is denoted Nb

(b) Gray (g) particles have range L ≥ 3 mm in emulsion. These tracks are mostly due to protons of kinetic energy in the range 26–400 MeV. Their multiplicity is denoted Ng. In each event, the black and gray tracks together are called heavily ionizing tracks. Their multiplicity is denoted Nh = Ng + Nb.

(c) Shower (s) particles are singly charged relativistic particles which mainly consist of pions. Their multiplicity is denoted Ns.

Event-by-event analysis demands the separation of events in to ensembles of collisions of different projectiles with hydrogen (H), light nuclei (CNO) and heavy nuclei (AgBr). Usually events with Nh ≤ 1 are classified as collision with hydrogen. Events yielding two to seven heavy tracks are classified as CNO events. Events with Nh ≥ 8 arise from collisions with heavy nuclei. In this method the separation of events for AgBr target is quite accurate in the sample with Nh ≥ 8 but in 2 ≤ Nh ≤ 7 there is an admixture of CNO events and peripheral collisions with AgBr target.
In this work, only events with a number of heavy tracks Nh ≥ 8 have been selected to exclude CNO interactions and peripheral collisions with AgBr target [11].

EXPERIMENTAL RESULT AND DISCUSSION

In Fig.1 (a-d), the MD of g-particles is fitted with a Gaussian distribution in the 32S-AgBr interactions at (a) 3.7A and (b) 200A GeV as well as in the 16O-AgBr interactions at (c) 60A and (d) 200A GeV. The comparison with the predictions from the Modified Fritiof Code (MFC) is performed as well in Fig. (1).

Table2: Experimental mean values for the g-particles and the corresponding calculated values using the Modified FRITIOF Code as well as the variance and the scaled variance (ratio of variance to mean).

Projectile	**Energy (A GeV)**	**$\langle N_g \rangle$**	**$\langle N_g \rangle_{th}$**	**Variance (σ^2)**	**$\sigma^2/\langle N_g \rangle$ ω**
^{32}S-AgBr	3.7	7.15±0.41	11.04	18.53±1.72	2.59±0.80
	200	4.75±0.23	9.87	9.04±1.63	1.90±0.36
^{16}O-AgBr	60	5.56±0.29	8.17	13.53±1.11	2.13±0.21
	200	5.01±0.30	8.25	9.61±1.29	1.92±0.28

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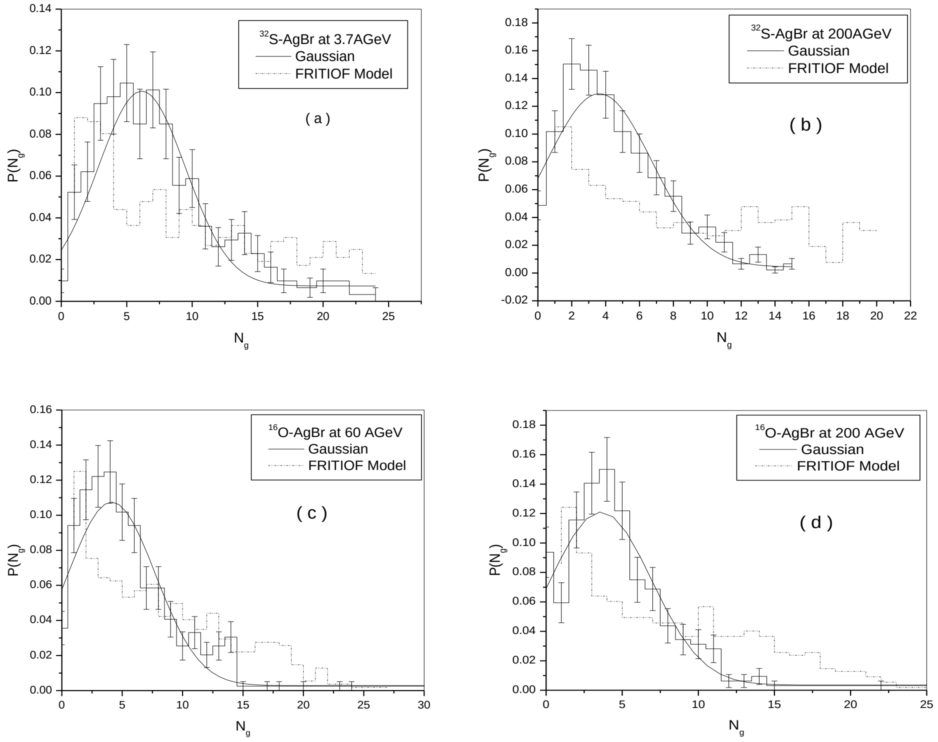

Fig. 1(a-d): Multiplicity distributions of fast target associated protons fitted with Gaussian distributions and Modified Fritiof code in the interactions of ^{32}S-AgBr (at3.7and 200A GeV) and ^{16}O-AgBr (at 60and 200A GeV).

Table 3: Parameters and $\chi 2$/DoF of the Gaussian fit of MDs for *g*-particles.

Projectile	**Energy (AGeV)**	**width**	**peak**	**Central value**	$\chi 2$	$\chi 2/\mathbf{DoF}$
^{32}S-AgBr	3.7	6.68±0.52	0.10	6.15±0.26	1.47	0.08
	200	6.25±0.46	0.13	3.62±0.22	1.65	0.11
^{16}O-AgBr	60	7.27±0.55	0.11	4.13±0.28	1.63	0.09
	200	6.66±0.59	0.12	3.60±0.31	1.08	0.07

The figures and tables show that the MDs of g-particles for all interactions have been fitted well with the Gaussian distribution function. While in general, the predictions of MFC in the region of Ng ≤ 10 underestimate the g-particle production and vice versa in the region of Ng >10.

To measure the multiplicity fluctuation among the emitted fragments, the investigated values of the scaled variance ω for all interactions in table (2) show that the MDs are not Poissonian at the lowest and highest available energies (ω > 1).

3. SCALING AND ENTROPY

Fig. (2a) shows the KNO scaling function $< n > P(n) = \Psi(Z)$ for g-particle multiplicity data at different energies fitted with the function

$\Psi(Z)$ = (1.66Z + 16.96Z3 -3.96Z5 + 1.68Z7- 0.09Z9) exp (-3.44Z) (8)

Fig. (2b) shows KNO scaling function $nP(n) = \varphi(Z)$ for g-particle multiplicity data at different energies fitted with the function

$\varphi(Z)$ = (1.01Z + 13.26Z3 -1.03Z5 + 1.57Z7- 0.10Z9) exp (-3.36Z) (9)

Investigating $nP(n)$ instead of $< n > P(n)$ has the obvious advantage that the statistical and systematic errors of <*n*> do not give contribution to the experimental uncertainty in the shape of the scaling function. Therefore $\varphi(Z)$ can be more selective between various theoretical predictions than$\Psi(Z)$.

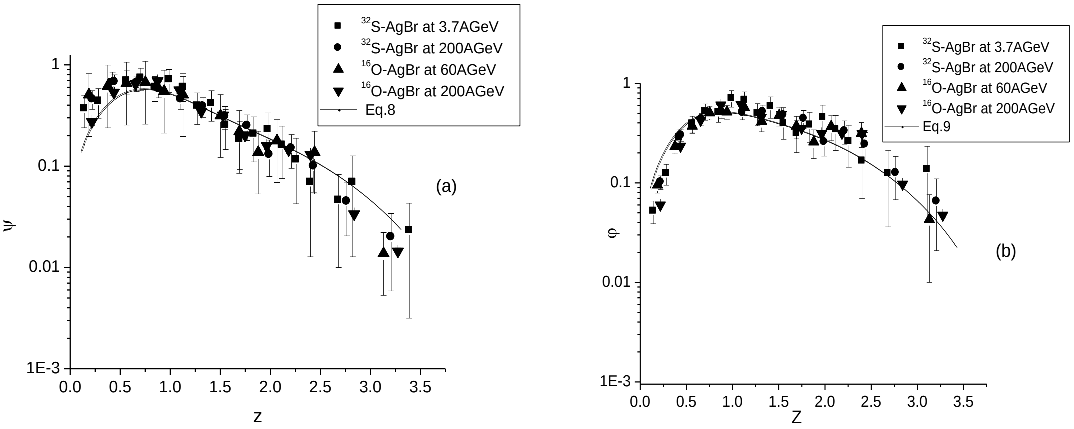

Fig. 2 (a, b): Multiplicity distributions of fast target associated protons in terms of KNO scaling in the interactions of 32S-AgBr (at3.7and 200A GeV) and 16O-AgBr (at 60and 200A GeV).

Because the gamma distribution has the scaling property (that is, if $\Psi(Z) \sim$ gamma(θ, k) with scale parameter θ and shape parameter k, then $\varphi(Z)$= Z $\Psi(Z)$also has the gamma distribution), Hegyi proposed a form for the scaling function using a new scaling variable $\omega = Z\,\theta$, which is given in Ref. 5 by Eq. (15)

$$\varphi\ (\omega) = \mathrm{N}.\ \omega k \exp(-\omega) \quad \text{with } \mathrm{N} = \Gamma^{-1}(k) \tag{10}$$

It is tempting to check its validity of Eq. (10) against the data shown in Figs. 2a and 2b. Accordingly, the solid curves in Figs. 3a and 3b represent the theoretical $\Psi(Z)$ and $\varphi(Z)$ respectively, corresponding to Eq. (10). The results lend support for the validity of the above scaling curve (Eq.10).

Obviously, the moments Mq of $\Psi(Z)$are independent of collision energy if KNO form is valid. Thus, the moments of the two KNO scaling functions are related by Eq. (11) in table (4).

$$M_q^{\varphi} = M_{q-1}^{\Psi} \tag{11}$$

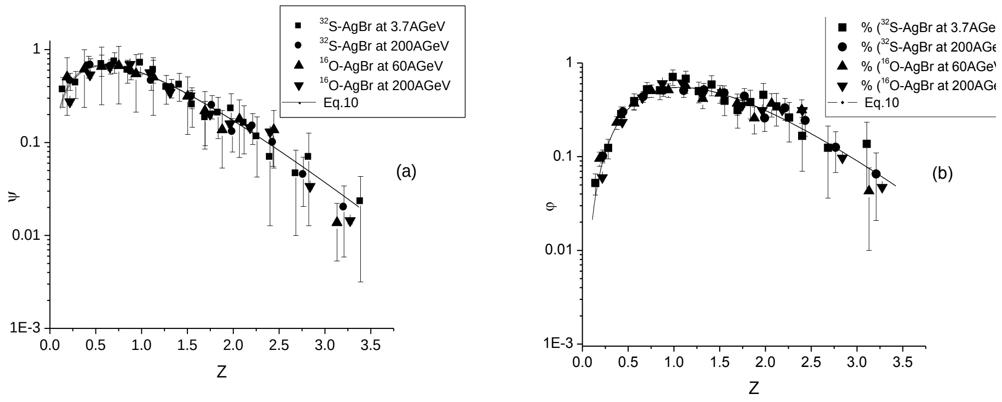

Fig.3 (a, b): Multiplicity distributions of fast target associated protons in terms of KNO scaling in the interactions of 32S-AgBr (at3.7and 200A GeV) and 16O-AgBr (at 60 and 200A GeV).

Table 4: Moments of the two scaling functions of Eq. (11).

Projectile	**Energy (A GeV)**	M_q^{Φ}				M_q^{Ψ}				
		M_2	M_3	M_4	M_5	M_2	M_3	M_4	M_5	M_6
^{32}S-AgBr	3.7	2.327	4.600	9.968	23.695	1.389	2.350	4.605	10.066	23.929
	200	2.700	5.741	13.525	34.318	1.547	2.838	6.034	14.216	36.072
^{16}O-AgBr	60	2.724	5.739	13.208	32.316	1.535	2.826	5.954	13.703	33.526
	200	2.628	5.464	12.572	31.097	1.596	2.869	5.965	13.725	33.948

In A-A collisions, entropy measurement may also serve as a tool to investigate the correlations and event-by-event fluctuations. However; we study here the energy dependence of multiplicity by estimating the entropy production in multiparticle systems, defined by Simak et al. [12] as

$$S = -\sum_n p_n \ln p_n \quad \text{where} \quad \sum_n p_n = 1 \qquad (12)$$

There exists a relationship between the entropy S, the average multiplicity$<n>$, and the KNO function$\Psi(Z)$, as long as the KNO form eq. (4) is valid. Therefore, the following inequality Eq. (13) is valid.

$$\frac{S}{ln < n >} \leq 1 + \frac{1}{ln < n >} \qquad (13)$$

Table 5: values of mean and entropy of g- particles prove the inequality of Eq. (13).

Projectile	**Energy (A GeV)**	**$< N_g >$**	**S**	**S/ ln$<N_g>$**	**1+1/ln$<N_g>$**
^{32}S-AgBr	3.7	7.152±0.411	2.735±0.157	1.390±0.080	1.508±0.087
	200	4.751±0.229	2.354±0.114	1.511±0.073	1.642±0.079
^{16}O-AgBr	60	5.560±0.287	2.568±0.133	1.461±0.075	1.569±0.081
	200	5.007±0.296	2.349±0.139	1.458±0.086	1.621±0.096

4. CONCLUSIONS

Based on the findings of this study, let us summarize our main results:

1- The multiplicity distributions for all interactions have been fitted well with the Gaussian distribution function. While in general, the predictions of MFC in the region of Ng ≤ 10 underestimate the g-particle production and vice versa in the region of Ng > 10.

2- The multiplicity distributions are broader than the Poisson distributions. The direct measure of the scaled variance ω, was used as a measure of multiplicity fluctuations. The measurements (ω >1) show that the production of target fragment at high energies cannot be considered as a statistically independent process.

3- We have demonstrated that besides $< n > P(n)$ the more simple combination $nP(n)$ also scales to a universal curve in the variable $n/$ <n> if KNO scaling holds valid.

4- A simplified universal function has been used in each scaling to display the experimental data.

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